

ber drawn from a uniform distribution in the range $[-1,1]$. This yields $\|n\| \leq \epsilon \leq 0.001 \|g_e\|$. Further, since $\|g_e\| \leq \|f\|$, we find $\omega = \epsilon \|f\|^{-1} \leq 0.001$. The value $g(x_j, y_k) = g_e(x_j, y_k) + n(x_j, y_k)$ is referred to as the blurred image. The blurred image is shown in FIG. 4.

Tikhonov restoration was applied to the blurred image using $\omega = 0.001$. The full restoration was contaminated by noise artifacts that made it impossible to visualize the image. See FIG. 5. A sequence of partial Tikhonov restorations was obtained at $t=0.35$, $t=0.25$, $t=0.15$, $t=0.005$, in addition to the full restoration at $t=0$. It will be understood that the partial restorations became sharper and noisier as $t \downarrow 0$. One may use the evolution as $t \downarrow 0$ to identify features that become obscured by noise at $t=0$. The growing influence of noise as $t \downarrow 0$ is a reflection of the $O(\epsilon^2)$ error bound characteristic of Tikhonov regularization as shown in Equation (18).

Image restoration method 10 was applied to the same blurred image, using $\omega=0.001$, $K=3$, and $s=0.01$. In view of the conservatively small value for s , the assumption that f satisfies Equation (13) is sound. Indeed, the full restoration is now found to be relatively free from noise, and to provide substantial improvement over the full prior art Tikhonov restoration. See FIG. 6. This is in accordance with Equation (19) in Theorem 2. The sequence of partial restorations at $t=0.35$, $t=0.25$, $t=0.15$, and $t=0.05$ in image restoration method 10 is likewise found to be better behaved than the corresponding prior art Tikhonov sequence.

The above experiment illustrates the preceding theoretical developments set forth with respect to image restoration method 10. At the same time, the experiment demonstrates the soundness of the computational implementation. Thus, image restoration method 10 may be reduced to practice as shown, for point spread functions in class G.

It is believed that image restoration method 10 may be advantageously applied in biomedical imaging, night vision systems, undersea imaging, imaging through the atmosphere, remote sensing, high definition television, as well as several other scientific and industrial applications where electron optics and class G point spread functions play a major role. Two key ideas make image restoration method 10 useful as a diagnostic tool in these and other fields. First, the substantial qualitative improvement in the full restoration that results from the additional constraint in Equation (13). Second, the display of the evolution of the restoration as t approaches 0. Together, these factors offer the possibility of greatly improved diagnostic capabilities, and provide a useful addition to current image restoration technology.

It is believed that method 10 is useful in this manner because it is based on mathematical tools which are different from prior art restoration algorithms. In contrast to the input-output linear system theory familiar to researchers with backgrounds in electrical engineering or computer science, and exemplified by Tikhonov restoration or Wiener filtering, method 10 is based upon partial differential equations, semi-group theory, and the mathematics of diffusion phenomena. The two key ideas mentioned in the preceding paragraph stem from this particular mathematical basis.

It will be understood that method 10 may be implemented on a general purpose computer programmed to perform the operations of blocks 12-22. Alternately method 10 may be performed using dedicated conventional hardware to perform such conventional opera-

tions as the transforms of blocks 14, 20. Additionally it will be understood by those skilled in the art that dedicated hardware may be provided, using conventional synthesis and fabrication techniques, for performing other operations within method 10.

I claim:

1. An image restoration method in a system having an image sensor and a digitizer, comprising the steps of:

(a) providing an imaging system described by an integral operator P ;

(b) transmitting an ideal image $f(x,y)$ through an image blurring and degrading transmission medium to provide a degraded image $g(x,y)$ wherein the degraded image $g(x,y)$ may be represented as $Pf=g$;

(c) receiving said degraded image $g(x,y)$ from said transmission medium by said image sensor;

(d) digitizing said degraded image $g(x,y)$ by said digitizer;

(e) transforming said degraded image $g(x,y)$ to provide a time modified representation $w(x,y,t)=P^s f$ wherein $w(x,y,0)=P^0 f$ represents the ideal image $f(x,y)$ at time $t=0$ prior to the operation of the integral operator P upon f and $w(x,y,1)=Pf=g$ represents the degraded image $g(x,y)$ when the image is received by said image sensor;

(f) requiring the magnitude of the difference between the ideal image f and a blurred version of f to be less than a preassigned tolerance value by minimizing $\|f-P^s f\|$ wherein $P^s f$ represents an image at time $t=s$ as s approaches 0;

(g) determining a plurality of values of $w(x,y,t)$ in accordance with said minimizing;

(h) adjusting said image in accordance with said determined plurality of values to provide a plurality of adjusted images;

(i) selecting an adjusted image of said plurality of adjusted images; and

(j) displaying said selected image.

2. The image restoration method of claim 1, comprising the further steps of:

displaying a sequence of partially restored images represented as $w(x,y,t)$ for a plurality of values of t as t approaches 0; and

determining an optimum image of said plurality of partially restored images in accordance with said displayed sequence.

3. The image restoration method of claim 1, wherein said constraining comprises the step of imposing the constraint $\|f-P^s f\| \leq K\epsilon$, where K is a constant and ϵ is representative of at least one image restoration parameter.

4. The image restoration method of claim 3, comprising the further step of imposing the constraint $\|Pf-g\| \leq \epsilon$, wherein ϵ is representative of at least one image restoration parameter.

5. The image restoration method of claim 4, comprising the further step of imposing the constraint $\|f\| \leq M$, wherein $M > \epsilon$.

6. The image restoration method of claim 5, comprising the further step of determining a restored image $f(x,y)$ which minimizes the quantity

$$\{\|Pf-g\|^2 + \|(\epsilon/M)f + (1/K)(f-P^s f)\|^2\}.$$

7. The image restoration method of claim 6, wherein a plurality of images $w(x,y,t)=P^s f$ are determined for a corresponding plurality of values of the time t .