deconvolution algorithms can be used to remove it as best as possible. For images, this degradation could be caused by motion blur, a partially blocked aperture, or an improperly focused lens system. The blur function d(x, y), also known as a point spread function (PSF), is representative of how a distant point of light travels through the optical system. [1] The mathematical representation of the blurred image g(x, y) is a convolution

$$g(x, y)=f(x, y)*d(x, y)+w(x, y) \tag{1}$$

where w(x, y) is a noise term, and * denotes the two-dimensional convolution. [1] The objective is to find the best estimate of f(x, y) from the noisy blurred image g(x, y) when d(x, y) is unknown. This relationship is simplified by transferring both sides of Equation 1 into frequency space via application of a Fourier transform. [9] This changes the convolution to multiplication, yielding

$$G(u, v)=F(u, v)D(u, v)+W(u, v) \tag{2}$$

where u and v are the coordinates of frequency space, and the capital letters represent the images in Fourier space. (Although one could achieve similar results with other transforms, the Fourier transform is the most common.) For the following discussion, I will assume noise is negligible.

The term 'deconvolution' is used to describe a process that removes the PSF from the image. There exists a large assortment of deconvolution algorithms. [1] The inverse filter is the most simple. The solution is

$$F(u, v) = \frac{G(u, v)}{D(u, v)}. \tag{3}$$

However, in most cases, the Fourier transform of the PSF D(u,v), also referred to as the optical transfer function (OTF), contains values that are very small. This results in a restored image f(x, y) that is dominated by noise.

To compensate for this, a constant is added to the denominator,

$$F(u, v) = \frac{G(u, v)D*(u, v)}{|D(u, v)|^2 + K}, \tag{4}$$

which is known as the pseudo-inverse filter (PIF). [1] The parameter K is typically chosen by trial and error. A smaller value increases the restored resolution, but also increases the noise. If the parameter is set too high, the image will not have changed. This filter is easy to employ and works well with most images.

If a least-squares estimation is applied to Equation 2, the result is

$$F(u, v) = \frac{G(u, v)D*(u, v)}{|D(u, v)|^2 + \frac{\sigma_N}{\sigma_S}} \tag{5}$$

where σ_N is the power spectral density (PSD) of the image noise, and σ_S is the PSD of F(u, v). (The PSD of a function A(u, v) is defined as $S_A(u)\ |A(u, v)|^2/NM$ where N and M are the dimensions of the image.) Equation 5 is known as the Wiener filter [10, 11, 12], named after Norbert Wiener. Since, F(u,v) is not known in empirical situations, σ_S must be approximated.

There exists many other algorithms that are capable of producing deconvolutions when the PSF is known. Some of these are described in reference [2]. The application of these methods is typically narrow in scope, producing good results within certain restrictions.

These algorithms require knowledge of the PSF to perform. However, in many situations the PSF is not measurable or cannot be easily modeled. To overcome this, a range of solutions has been developed known as blind deconvolution algorithms. Blind deconvolution is a general term describing techniques that remove aberrations caused by some unwanted low-pass filtering technique where the PSF is not determined. Blind deconvolution techniques can be either iterative [4, 5, 6, 7] or non-iterative [8, 13]. Iterative techniques, which comprise of the majority of blind deconvolution techniques, are based on equations that require multiple applications. After many iterations, the program converges on a solution if the parameters are set correctly. As such, they generally require a significant amount of computation and can be difficult to implement. [13] Often, both classes require that at least a certain amount of information about the degradation be known.

An optimum algorithm would then be non-iterative, easy to implement, able to handle significant noise, and effectively operate on a large class of images. To the best of my knowledge, SeDDaRA is the first algorithm to meet such requirements.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a flowchart showing the primary steps for the extraction of the filter function and the deconvolution of a data set.

FIG. **2**A is an image of the planet Saturn, taken by the Hubble telescope.

FIG. **2**B is the Fourier transform of the image. FIG. **2**C shows the transform after the phase information is discarded, and a smoothing function applied.

FIG. **2**D is the deconvolution of the image using the extracted point spread function, shown in FIG. **2**E.

FIG. **3**A was taken from the space probe Galileo of the surface of the moon Io of Jupiter. FIG. **3**B shows the restoration result after applying the algorithm with a frequency-dependent α.

FIG. **4**A was a photograph taken by a standard camera and digitized.

FIG. **4**B is the restoration of the image using the SeDDaRA process.

FIG. **5**A shows an ultrasound waveform that traveled through a centimeter of air and its restoration. FIG. **5**B shows the frequency spectra of the two waves.

SeDDaRA THEORY

General Formulation

The theoretical basis for the technique is still being studied. Although this work continues, a basic understanding has been established. Essentially, the process begins with Equation 2 where only G(u, v) is known. Using this process, D(u, v) is estimated and F(u, v) is found using a deconvolution algorithm. So, as a pure analytic calculation, there is one equation and two unknowns, and the process finds both. Mathematically this is impossible. In order for a blind deconvolution technique to work, some information about the either the original data or the PSF must be known.

Instead of estimating either function, SeDDaRA assumes that there is a relationship between the smoothed magnitude of the truth image and the PSF represented as

$$D^{1-\alpha}(u, v)=K_D S\{F^{\alpha}(u, v)\} \tag{6}$$