$$\hat{m}_{n+1}=\hat{\nabla}_{-1}\hat{W}_{e(n+1)}\hat{m}_{g,n+1}{}^{w}.$$

Another important new element of this invention is introducing the penalization function in the image restoration process.

Let us assume that the ideal image can be described as a combination of the background image, M_{bg} (r), and the anomalous part of the image, $m_{\alpha}(r)$, where r is the coordinate of some point in domain V. In this situation it is required that there should be only two values of the parameter m(r) in the focused image equal to the background value or to the anomalous value. However, the geometrical distribution of these values is unknown. We can force the inversion to produce an image which not only generates the observed degraded image but which is also described by these known values, thus painting the geometry of the ideal object. We call this approach "penalization". There is a simple and straightforward way of combining penalization and the MGS method. Numerical tests show that MGS generates a stable image, but it tends to produce the smallest possible anomalous domain. It also makes the image look unrealistically sharp. At the same time, the image parameter values m(r) outside this local domain tend to be equal to the background value $m_{bg}(r)$ which nicely reproduces the background image. We can now impose the upper bound for the positive anomalous parameter values $m_a{}^{ub}(r)$, and during the sharpening process simply chop-off, or discard, all the values above this bound. This algorithm can be described by the following formula:

$$m(r)-m_{bg}(r)=m_{ub}{}^{a}(r),\ \text{if}\ [m(r)-m_{bg}(r)]\geqq m_{ub}{}^{a}(r):$$

$$m(r)-m_{bg}(r)=0,\ \text{if}\ [m(r)-m_{bg}(r)]\leqq 0.$$

Thus, according to the last formula the image parameters values m(r) are always distributed within the interval:

$$m_{bg}(r)\leqq m(r)\leqq m_{bg}(r)+m_{ub}{}^{a}(r).$$

A similar rule is applied for the case of negative anomalous parameter values.

Hence, an important feature of the present invention is the ability to enhance and sharpen degraded images. It is realized numerically as the following seven steps.

Step 1. Constructing the initial restoration of the ideal image gradient parameters $\hat{m}_{g,1}$ according to the formula

$$\hat{m}_{g,1}=-k_0\hat{l}(\hat{m}_0{}^{w})=-k_0\hat{W}_{e0}{}^{-1}\hat{B}_g{}^{*}(\hat{B}_g(\hat{W}_{e0}{}^{-1}\hat{m}_{g,0}{}^{w})-\hat{d})-\alpha k_0\hat{m}_{g,0}{}^{w} \quad (43)$$

Note that we assume that starting zero iteration is equal to zero $\hat{m}_{g,0}{}^{w}=0$, and $\hat{W}_{e0}$ is equal to the identity matrix $\hat{W}_{e0}=\hat{I}$, so the starting approximation of the ideal image gradient parameters is generated by applying the transposed complex conjugated blurring operator and the inverse gradient operator to the blurred image. Mathematically it can be described by matrix multiplication:

$$\hat{m}_{g,1}=k_0\hat{B}_g{}^{*}\hat{d}=k_0\hat{\nabla}^{-1}\hat{B}^{*}\hat{d}_1 \quad (44)$$

where k_0 is the length of the initial step.

Step 2. Computing the inverse sharpening filter (matrix $\hat{W}_{e1}{}^{-1}=\hat{M}^{-1}{}_{e}(\hat{m}_{g,1})$ on the 1-st iteration as the matrix of the function $w_e(m_{g,1})$ introduced above in equation (27):

$$w_e(m_{g,1})=(\|m_{g,1}\|^2+e^2)^{1/2}\sim\|m_{g,1}\|. \quad (45)$$

Step 3. Constructing the partially sharpened image (second iteration to the weighted restored image) according to the formula:

$$\hat{m}_{g,2}{}^{w}=\hat{m}_{g,1}+\delta\hat{m}_{g,1}{}^{w}=\hat{m}_{g,1}-k_1\hat{l}(\hat{m}_{g,1}{}^{w}),$$

Here

$$\hat{l}(\hat{m}_{g,1}{}^{w})=\hat{l}(m_{g,1}{}^{w})+\beta_1\hat{l}(\hat{m}_{g,0}{}^{w}).$$

where

$$\hat{l}(\hat{m}_{g,1}{}^{w})=\hat{l}(m_{g,0}{}^{w})=W_{e0}{}^{-1}B_g{}^{*}(\hat{B}_g(\hat{W}_{e0}{}^{-1}\hat{m}_{g,0}{}^{w})-d)+\alpha\hat{m}_{g,0}{}^{w},$$

$$\hat{l}(m_{g,1}{}^{w})=W_{e1}{}^{-1}B_g{}^{*}(\hat{B}_g(\hat{W}_{e1}{}^{-1}\hat{m}_{g,1}{}^{w})-d)+\alpha\hat{m}_{g,1}{}^{w}.$$

The length of the step k_1 is determined by the line search to the minimum of the functional $P_g{}^{0}$ $(\hat{m}_{g,1}{}^{w}-k\hat{l}(\hat{m}_{g,1}{}^{w}))$ Parameter β_1 is calculated by the formula:

$$\beta_1=\frac{\|\hat{l}(\hat{m}^{\omega}_{g,l})\|^2}{\|\hat{l}(\hat{m}^{\omega}_{g,0})\|^2}.$$

Step 4. Computing the inverse sharpening filter (matrix $\hat{W}_{en}$) on the n-th iteration (n=2,3, . . .) as the matrix of the weighting function $w_e(m_{g,n-1})$ introduced above in equation (27):

$$w_e{}^{-1}(m_{g,n-1})=(\|m_{g,n-1}\|^2+e^2)^{1/2}\sim\|m_{g,n-1}. \quad (46)$$

Step 5. Constructing the partially sharpened weighted image ((n+1)-th iteration to the restored image) according to the formula

$$\hat{m}_{g,n+1}\sim\hat{m}_{g,n}{}^{w}+\delta\hat{m}_{g,n}{}^{w}=\hat{m}_{g,n}{}^{w}-k_n\hat{l}(\hat{m}_{g,n}{}^{w}), \quad (47)$$

where weighted conjugate gradient directions are determined by the expressions:

$$\hat{l}(\hat{m}_{g,n}{}^{w})=\hat{l}(\hat{m}_{g,n}{}^{w})+\beta_n\hat{l}(\hat{m}_{g,n-1}{}^{w}), \quad (48)$$

$$\hat{l}(\hat{m}^{w}{}_{g,n})=\hat{W}_{en}{}^{-1}\hat{B}_g{}^{*}(\hat{B}_g(\hat{W}_{en}{}^{-1}\hat{m}^{w}{}_{g,n})-\hat{d})+\alpha\hat{m}^{w}{}_{g,n}. \quad (49)$$

The length of the step k_n is determined by the line search to the minimum of the functional P^{α} $(\hat{m}_{g,n}{}^{w}-k_n\hat{l}(\hat{m}_{g,n}{}^{w}))$. Parameter β_n is calculated by the formula:

$$\beta_n=\frac{\|\hat{l}(\hat{m}^{\omega}_{g,n})\|^2}{\|\hat{l}(\hat{m}^{\omega}_{g,n-1})\|^2}.$$

Step 6. Recomputing the real parameters of the ideal image from the weighted parameters at the (n+1)-th iteration by inverse filtering the weighted image:

$$\hat{m}_{n+1}=\hat{\nabla}^{-1}\hat{W}_{e(n+1)}{}^{-1}\hat{m}_{g,n+}{}^{w} \quad (50)$$

Step 7. Imposing the upper bound $m_a{}^{ub}$ (r) for the positive anomalous parameter values, and the lower bound $-m_a{}^{1b}$ (r) for the negative anomalous parameter values and chopping-off, or discarding, all the values outside these bounds during entire image restoration process to keep the image parameters values m_n (r) always distributed within the interval:

$$m_{bg}(r)-m_a{}^{1b}(r)\leqq m_n(r)\leqq m_{bg}(r)+m_a{}^{ub}(r). \quad (51)$$

In the drawing, FIG. **1** presents the flow chart illustrating the image enhancement and sharpening method 10 of the present invention. The method can be performed upon the degraded image expressed as a digitized M×M array d=d (x_i,y_j)(for a 2-D image) or a digitized M×M×M array d=d (x_i, y_j, z_k) (for a 3-D image). The linear blurring operator $\hat{B}$ is assumed to be given as a square matrix of the size $M^2\times M^2$ (for a 2-D image) or $M^3\times M^3$ (for a 3-D image).

Referring again to FIG. **1** the first step is the initial restoration of the image (block **11**) by applying a transposed