tude **150** with high efficiency and, furthermore, with a high optical throughput (i.e., low loss). Once phase functions ξ and ϕ are determined, the respective DOEs will be fabricated to impose these respective phase maps onto the incident and output beams, via the respective transmission coefficients, $e^{i\xi}$ and $e^{i\Phi}$, as shown in FIG. **1**.

Since the respective optical fields, E_1 and E_2, at the respective planes containing DOE-**1** and DOE-**2**, are related via a spatial. Fourier transform, it is possible to use various classes of phase retrieval computational methods to ascertain the necessary phase profiles, ξ and ϕ, at the two planes. In this manner, the respective DOEs can be fabricated, each with the respective phase-plate profile, ξ and ϕ, that results in a self-consistent solution.

For the purposes of simulations and experimental demonstrations, and, without loss of generality, we consider the implementation of a mode-conversion module that enables high-efficiency coupling of a single-mode fiber laser oscillator output beam to a specific HOM of a fiber power amplifier. As discussed in more detail below, the ribbon fiber amplifier is fabricated so that a single spatial mode exists along one of the cross-sectional dimensions (the x-axis), and, that a higher-order mode (HOM) exists along the orthogonal cross-sectional dimension (the y-axis). In one example, the HOM of the fiber amplifier, to which one desires to launch the seed laser output, is chosen to be the 7^{th} order eigenmode of a rectangular-core fiber (i.e., a ribbon fiber).

FIG. **2** shows an example of a step-index ribbon fiber **200** wherein the refractive index of the cladding, n_{clad}, **210** is slightly smaller relative to the refractive index of the rectangular core, n_{core}, **220**, or, guiding region. In this example, the cross-sectional dimension of the guiding region **220** is 5 μm in the x-direction and 50 μm in the y-direction. For this choice of dimensions and refractive indices, the guided spatial mode is assumed to be a single mode in the x-direction and the desired. HOM on the ribbon fiber is assumed to be the 7^{th} order spatial mode in the y-direction.

Phase Retrieval via Gerchberg-Saxton Algorithms

The prior art includes various mathematical algorithms with the capability to determine the phasefront (or, wavefront) profile of a complex electromagnetic field at a given pair of planes in space, given the respective amplitudes of the field (i.e. the magnitude of the complex field) at the respective planes. The general technique is commonly referred to in the art as “phase retrieval.” One such mathematical construct is known in the art as the Gerchberg-Saxton (G-S) algorithm, which is utilized in the design rules for the mode-converter described herein.

The G-S algorithm involves an iterative process that enables one to determine the phase profiles of a field, at two different planes in space, given the amplitudes of the given field at the two said planes. We shall assume that the functions involved are well-behaved in a mathematical sense and that the iterative process converges to a single stable state. Typically, the two specified planes in space are related by a Fourier transform (e.g., the near-field and the far-field locations). Under these conditions, the G-S algorithm proves a means for phase retrieval of the field, in a self-consistent manner.

In the context of this invention, it is assumed that a pair of optical field amplitudes is specified, with the goal to determine the phase map of each field, at their respective Fourier transform planes, that results in an self-consistent iterative solution. Suffice it to say, the criteria for convergence of the algorithm has various interpretations, such as, meeting or exceeding a predetermined threshold in terms of the correlation, overlap integral or other comparative metric that compares the n^{th} iteration with the $(n+1)^{th}$ iteration.

There are myriad variants on prior-art G-S algorithm. Referring to FIG. **3**, a flow chart, **300**, depicting one variant of G-S is shown. In this approach, there are two constraints, namely, the amplitude of an EM field at each conjugate transform plane. The corresponding phase map of each electromagnetic (EM) field at its respective transform plane is allowed to vary freely, subject to the iterative system satisfying the pair of transform-related constraints. In what follows, we assume that the slowly varying envelope of the EM wave is a complex quantity, comprised of an amplitude and a phase function, and, further, that the field is monochromatic.

In this example, the starting point of the G-S algorithm corresponds to specifying the amplitude in the near-field output plane of the single-mode fiber oscillator. The near-field intensity is given by $|g|^2$; the far-field intensity is given by $|G|^2$, which, for future reference, also corresponds to the near-field of the ribbon fiber amplifier; and the Fourier transform of g is given by G, thereby forming a conjugate pair of transform variables. Hence, the respective field amplitudes, $|g|$ and $|G|$, form the two constraints. Turning now to FIG. **3**, and, given these definitions and constraints, the four-step G-S iterative approach is implemented as follows:

Step 1: The amplitude of the BM field, $g=|g|e^{i\xi}$, at DOE-**1** (recall, FIG. **1**), is constrained, to be a specific function, $|g|$, **310**, whereas, the phase, ξ of the field **315** is unconstrained. The phase will tend to a self-consistent function upon convergence of the algorithm. In this case, the initial amplitude **310** is constrained to be the desired near-field single-mode (LP_{01} or TEM_{00}) output of the seed (master) oscillator.

Step 2: The complex field, $g=|g|e^{i\xi}$ is spatially propagated, from the plane containing DOE-**1**, **110** (located at the front focal plane of the lens **130**, as shown in FIG. **1**), through the lens **130** and, finally, evaluated at the rear focal plane of the same lens, namely, at the plane containing DOE-**2**, **120**. In the Fraunhofer approximation, which is applicable for these practical systems, it is well-known that the EM complex field at the rear focal plane of a lens will be proportional to the spatial Fourier transform of the incident field at the front focal plane, that is, $G \propto \mathcal{F}\{g\}$, where $\mathcal{F}$ is the spatial Fourier transform operator. The resultant complex field, $G=|G'|e^{i\Phi}$, at the rear focal plane is comprised at an amplitude, $|G'|$, **320** and a phase function (ϕ) **325**.

Step 3. The resultant Fourier-transformed amplitude ($|G'|$), **320** is replaced by the fixed amplitude function ($|G|$) **330**. In the context of the mode converter, $|G|$ corresponds to the desired HOM amplitude of the desired ribbon fiber amplifier eigenmode. Hence, the complex field $G=|G'|e^{i\Phi}$ becomes $G=|G|e^{i\Phi}$. Recall, that this field amplitude is the second of the two constraints inherent in the G-S algorithm. The phase function (ϕ) **325**, as determined by the Fourier transform operation of Step 2, is unconstrained, as shown by the phase function **335**.

Step 4. The complex field, $G=|G|e^{i\Phi}$, is then spatially propagated, from its position at DOE-**2**, **120**, that is, the rear focal plane of the lens, back through the lens **130** and, finally, evaluated at the front focal plane of the same lens, namely, at the plane containing DOE-**1**, **110**. The lens **130** generates the Fourier transform of the complex field, $G=|G|e^{i\Phi}$, resulting in a field $g=|g'|e^{i\xi}$ at DOE-**1**. The EM field $g=|g'|e^{i\xi}$ is comprised of amplitude ($|g'|$) **340** and phase function (ξ) **345**, respectively. The final operation of this initial iteration involves replacing the amplitude ($|g'|$) **340** with the amplitude ($|g|$) **310** at the plane containing the element DOE-**1**. This amplitude ($|g|$) corresponds to the first constraint of the G-S algorithm, as indicated in Step 1 above. As before, the phase function (ξ) **345** is unconstrained, as shown by **315**. This completes the first iteration of the phase retrieval algorithm.