

environmental or surveillance satellites, in the detection of deep-space satellites from ground-based telescopes, and in observational astronomy (see J. C. Wyant, Ed. *Imaging Through the Atmosphere*, Proceedings of the Society of Photo-Optical Instrumentation Engineers, Vol. 75 (1976); N. S. Kopeika, 'Spectral Characteristics of Image Quality for Imaging Horizontally Through the Atmosphere,' *Applied Optics*, Vol. 16 (1977), pp. 2422-2426; R. Weber, 'The Ground-Based Electro-Optical Detection of Deep-Space Satellites,' *Applications of Electronic Imaging Systems*, Proceedings of the Society of Photo-Optical Instrumentation Engineers, Vol. 143, R. E. Franseen and D. K. Schroder, Eds. (1978), pp. 59-69; N. S. Kopeika, 'Imaging Through the Atmosphere for Airborne Reconnaissance,' *Optical Engineering*, Vol. 26 (1987), pp. 1146-1154; J. D. Gonglewski and D. G. Voelz, 'Laboratory and Field Results in Low Light Post-detection Turbulence Compensation Using Self Referenced Speckle Holography,' *Digital Image Synthesis and Inverse Optics*, Proceedings of the Society of Photo-Optical Instrumentation Engineers, Vol. 1351, A. F. Gmitro, P. S. Idell, and I. J. LaHaie, Eds. (1990), pp. 798-806.

The case $\beta=1/2$ corresponds to the Cauchy or Lorentzian density, and may be used to describe X-ray scattering in radiology (see F. C. Wagner et al., 'A Characterization of the Scatter Point Spread Function in Terms of Air Gaps,' *IEEE Transactions on Medical Imaging*, Vol. 7 (1988), pp. 337-344).

A wide variety of electron-optical devices obey Equation (5) with a value of B satisfying $1/2 \leq \beta \leq 1$ (see C. B. Johnson, 'A Method for Characterizing Electro-Optical Device Modulation Transfer Functions,' *Photographic Science and Engineering*, Vol. 14 (1970), pp. 413-415; C. B. Johnson, 'Classification of Electron-Optical Device Modulation Transfer Function,' *Advances in Electronics and Electron Physics*, Vol. 33B (1972), pp. 579-584; C. B. Johnson et al., 'High-Resolution Microchannel Plate Image Tube Development,' *Electron Image Tubes and Image Intensifiers II*, Proceedings of the Society of Photo-Optical Instrumentation Engineers, Vol. 1449, I. P. Csorba, Ed. (1991), pp. 2-12).

Such devices constitute important elements of various biomedical imaging modalities, including image intensifier-video camera (II-TV) fluoroscopic systems (see S. Rudin et al., 'Improving Fluoroscopic Image Quality with Continuously Variable Zoom Magnification,' *Medical Physics*, Vol. 19 (1991), pp. 972-977); radiographic film digitizers (see F. F. Yin et al., 'Measurement of the Presampling Transfer Function of Film Digitizers Using a Curve Fitting Technique,' *Medical Physics*, Vol. 17 (1990), pp. 962-966); radiographic selenium imaging plates (see P. J. Papin and H. K. Huang, 'A Prototype Amorphous Selenium Imaging Plate System for Digital Radiography,' *Medical Physics*, Vol. 14 (1987), pp. 322-329); computed radiography photostimulable phosphor systems (see S. Sanada et al., 'Comparison of Imaging Properties of a Computed Radiography System and Screen-Film Systems,' *Medical Physics*, Vol. 18 (1991), pp. 414-420; H. Fujita et al., 'A Simple Method for Determining the Modulation Transfer Function in Digital Radiography,' *IEEE Transactions on Medical Imaging*, Vol. 11 (1992), pp. 34-39); digital TV tomography systems (see M. Takahashi et al., 'Digital TV Tomography: Description and Physical Assessment,' *Medical Physics*, Vol. 17 (1990), pp. 681-685); and radiographic screen-film systems (see Y. Higashida et al., 'Dual-Film Cassette Technique for Studying the Effect of Radiographic Image Quality on Diagnostic Accuracy,' *Medical Physics*, Vol. 11 (1984), pp. 646-652; H.

Kuhn and W. Knupfer, 'Imaging Characteristics of Different Mammographic Screens,' *Medical Physics*, Vol. 19 (1992), pp. 449-457).

Other important image tube/image intensifier applications include high definition television (HDTV) (see N. V. Rao, 'Development of a High-Resolution Camera Tube for 2000-Line TV System,' *Electron Tubes and Image Intensifiers*, Proceedings of the Society for Photo-Optical Instrumentation Engineers, Vol. 1243, I. P. Csorba, Ed. (1990), pp. 81-86; R. Barden et al., 'High Resolution MS-Type Saticon Pick-Up Tube with Optimized Electron-Optical Properties,' *Electron Tubes and Image Intensifiers II*, Proceedings of the Society for Photo-Optical Instrumentation Engineers, Vol. 1449, I. P. Csorba, Ed. (1991), pp. 136-147, and night vision and undersea imaging systems (see K. A. Costello et al., 'Imaging GaAs Vacuum PhotoDiode with 40% Quantum Efficiency at 530 nm,' *Electron Tubes and Image Intensifiers*, Proceedings of the Society for Photo-Optical Instrumentation Engineers, Vol. 1243, I. P. Csorba, Ed. (1990), pp. 99-106).

In a typical imaging situation, several electron-optical devices may be cascaded together and used to image objects through a distorting medium such as the atmosphere or the ocean. The overall optical transfer function is then the product of finitely many functions of the type given by Equation (5), i.e.,

$$\beta(\xi, \eta) = e^{-\sum_{i=1}^n (\lambda_i \xi^2 + \eta^2)^{\beta_i}}, \quad \lambda_i \geq 0, \quad 0 < \beta_i \leq 1. \quad \text{Equation (5A)}$$

Problems having optical transfer functions satisfying Equation (5A) may be referred to as space-invariant image restoration problems wherein the image restoration problem is equivalent to an ill-posed backwards in time continuation problem for a generalized differential equation. Each factor in this product represents a component of the total blur, and is described by Equation (5) with a particular value for λ and β . A method for determining λ_i and β_i for each component is discussed in C. B. Johnson, 'Classification of Electron-Optical Device Modulation Transfer Function,' *Advances in Electronics and Electron Physics*, Vol. 33B (1972), pp. 579-584. Equation (5A) also describes cascaded electron-optical devices forming the imaging chain in many digital radiography systems. Frequency domain characterization of such systems is an important and ongoing task, as is evident from the above-cited references. In several industrial applications, the general functional form defined by Equation (5A) may be found to best-fit an empirically determined optical transfer function, by suitable choices of the parameters λ_i , β_i and J.

It is emphasized that while many imaging phenomena do not have optical transfer functions that can be well-described by Equation (5A), the latter nevertheless encompasses a significant set of imaging problems. We denote by G the class of optical transfer functions defined by Equation (5A). Note that Equation (5) is a special case of Equation (5A). The functions in G are infinitely divisible probability density functions.

Most current approaches to image restoration are based on linear system inverse filter theory, where the input is the degraded image, and the single output is an approximation to the ideal image. Using a priori constraints on the unknown ideal image, various regularization methods are employed in these approaches, in order to stabilize the deconvolution procedure in the presence of noise. Tikhonov regularization is one of the best-known examples of such procedures. This method is closely related to Wiener filtering. The Tikhonov regularization method is also known as constrained least