

$$z'1 \begin{cases} a1 = -a1.c1 \\ \beta1 = -b1.c1 \\ \gamma1 = 1 - c1^2 = a1^2 + b1^2 \end{cases}$$

The projection  $\vec{B}'1$  of  $\vec{B}$  on the horizontal plane defined by  $\vec{z}'1$  may be written as:

$$\vec{B}'1 = \vec{B} + \vec{z}'1.B \cos A$$

because B and Z1 are opposite in direction, i.e.:

$$\vec{B}'1 \begin{cases} Bx + a1.B \cos A \\ By + b1.B \cos A \\ Bz + c1.B \cos A \end{cases}$$

The oriented angle ( $\vec{B}'1, \vec{z}'1$ ) that be defined, for instance, by its cosine or its sine sign, will now be calculated.

Here the sign of the sine will be given by the sign of the vectorial product  $\vec{B}'1 \wedge \vec{z}'1$ , along axis OZ1. Then

$$\cos |\vec{B}'1, \vec{z}'1| = \frac{a1.(Bx + a1.B \cos A) + \beta1.(By + b1.B \cos A) + \gamma1.(Bz + c1.B \cos A)}{||\vec{z}'1|| ||\vec{B}'1||}$$

with:  $||\vec{B}'1|| = B \cdot \sin A$  and  $||\vec{z}'1|| = (a1^2 + b1^2)^{\frac{1}{2}}$  from which, all the calculations being made and equation (2) being taken into account, it results:

$$\cos |\vec{B}'1, \vec{z}'1| = \frac{Bz + c1.B \cos A}{B \cdot \sin A \cdot (1 - c1^2)^{\frac{1}{2}}}$$

Furthermore:

$$\begin{aligned} \text{sign} (\vec{B}'1 \wedge \vec{z}'1) &= \text{sign} [\beta1(Bx + a1.B \cos A) - \\ &\quad a1.(By + b1.B \cos A)] \\ &= \text{sign} [\lambda1.c1.(Gx.By - Gy.Bx)] \end{aligned}$$

It will be noted that the cosine is also simply expressed as:

$$\begin{aligned} \cos (\vec{B}'1, \vec{z}'1) &= + \frac{Bz + c1.B \cos A}{\lambda1.G \alpha.B \sin A} \text{ when } \lambda1 > 0 \\ &= - \frac{Bz + c1.B \cos A}{\lambda1.G \alpha.B \sin A} \text{ when } \lambda1 < 0 \end{aligned}$$

The value of d may then be derived from the above calculations, as above mentioned.

It is obviously necessary to elect, between the two roots of equation (5) the one that gives the correct vertical line of the anchoring spot. Strictly speaking, it is not possible to make the selection from only the measured parameters, without introducing other physical considerations by another measurement, for instance the measurement of the wind.

However, preliminary observations allow the invention to distinguish between the two roots from the parameter L.

Indeed,  $L = (B \cdot \cos A) \cdot Bz^{-1}$  is always negative when the swell slope remains lower than ( $90^\circ - A$ ), which would correspond the the case of a not very rough sea.

Furthermore, with  $L = L'$  and when  $L' < 1$ , then:

$$\lambda1\lambda2 = - \frac{1 - L^2}{G\alpha^2 + K^2} < 0$$

5 when  $\beta1 = |\vec{B}'1, \vec{z}'1|$  corresponds to the first root and  $\beta2 = |\vec{B}'2, \vec{z}'2|$  to the second root, one can verify that for the two roots the vertical speed  $Vz$  have opposite signs and an election may be made of:

$$10 \quad d1 = -\beta1$$

and

$$d2 = -(\beta2 + \pi)$$

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with in any case,  $d1 + d2 = \pi$ .

When  $L' > 1$ , then  $\lambda1.\lambda2 > 0$ ,  $\lambda1$  and  $\lambda2$  both having the sign of K. One can verify that p1 and p2 have the same sign, as well as the vertical speeds  $Vz1$  and  $Vz2$ .

20 Furthermore:

$$d1 = -\beta1$$

$$d1 = -\beta1 - \pi$$

or

$$d2 = -\beta2 = +\beta1 - \pi \quad d2 = -\beta2 - \pi = 1$$

Now when  $\lambda1 > \lambda2$ , it may be positively stated, by continuity when the correct root is  $\lambda1$ , that root remains correct until the equation (5) has a double root, i.e. for:

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$$(KL)^2 = -(1 - L^2) \cdot (G\alpha^2 + K^2)$$

Such a condition is possible only when  $L' > 1$ .

On the other hand, it is known that the vertical speed must be substantially nil when the buoy itself is vertical, i.e. on either a crest or a trough.

Therefore the buoy is vertical when the true vertical line OZi is in coincidence with Oz. At that time:

$$L = B \cdot \cos A \cdot Bz^{-1} = -1.$$

Conversely, there are two roots when  $L = -1$ :

$$\lambda_i = 0, \text{ and}$$

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$$\lambda_j = \frac{2K}{G\alpha^2 + K^2}$$

$\lambda_j$  having the sign of K. When  $\lambda_i$  is the good root, Ozi is the exact vertical line on the spot; when  $\lambda_j \neq 0$  is the good root, Ozj is the second intersection of the cone (B, A) with the vertical plane containing the direction of the swell.

Thus it may be understood that, in the cycle of a wave, the parameter L may vary according two different modes:

mode No. 1	
phase 0+	$L' > 1$ , Oz is inclined towards the South sector
	$L' = 1$ , Oz is crossing the vertical line,
	$L' < 1$ , Oz is inside the cone (B, A)
phase 0-	$L' = 1$ , Oz is crossing the vertical line in the opposite direction.
mode No. 2	
	$L' > 1$ , Oz is inclined towards the South sector,