

(2) $d = -(B', z') - \pi$, when $V_Z > 0$, because the bearings are counted along the direction opposite to the trigonometric direction in the horizontal plane OX, OY.

The sign of V_Z being known, the sign indeterminations on p and π of on d are cleared, those indeterminations being unavoidable since there is no privileged direction in the arrangement of the accelerometers and the magnetometers.

The calculations of the coordinates a, b, c of the vector \vec{Z} in the trihedral Ox, Oy, Oz will now be described.

The following equations result:

$$a^2 + b^2 + c^2 = 1 \tag{1}$$

$$\frac{-\vec{Z} \cdot \vec{B}}{B} = \cos A \text{ because } (-\vec{Z}, \vec{B}) = A,$$

where A is the constant angle between the vertical line of the spot and the earth magnetic field, B is the constant modulus of the magnetic field, A and B depending on the anchoring spot only.

Furthermore, \vec{Z}, \vec{G} and \vec{z} being coplanar, then:

$$\vec{Z}(\vec{G} \wedge \vec{z}) = 0.$$

This equation when developed gives, with (1):

$$a^2 + b^2 + c^2 = 1 \tag{1}$$

$$a.Bx + b.Bx + c.Bz = -B \cdot \cos A \tag{2}$$

$$a.Gy - b.Gx = 0 \tag{3}$$

resulting in:

$$a = \lambda \cdot Gx \tag{4}$$

$$b = \lambda \cdot Gy$$

$$c = -\frac{B \cdot \cos A}{Bz} - \lambda \cdot \frac{Bx \cdot Gx + By \cdot Gy}{Bz}$$

The values a, b, c of the equation (1) being replaced by their values in the system (4), the solution of the hereunder equation (5) gives the parameter λ :

$$0 = \lambda^2 \cdot \left[(Gx^2 + Gy^2) + \left(\frac{Bx \cdot Gx + By \cdot Gy}{Bz} \right)^2 \right] +$$

$$2\lambda \cdot \frac{B \cdot \cos A (Bx \cdot Gx + By \cdot Gy)}{Bz^2} - \left(1 - \frac{B^2 \cdot \cos^2 A}{Bz^2} \right)$$

With:

$$Go = (Gx^2 + Gy^2)^{\frac{1}{2}}$$

$$K = (Bx \cdot Gx + By \cdot Gy) \cdot Bz^{-1}$$

$$L = B \cdot \cos A \cdot Bz^{-1}$$

equation (5) becomes:

$$\lambda^2 (Go^2 + K^2) + 2\lambda LK - (1 - L^2) = 0$$

and then:

$$c = -(L + \lambda K)$$

The λ equation (5) is thus a quadratic equation which has two roots: $\lambda 1$ et $\lambda 2$, defining two vectors $\vec{Z}1(a1, b1, c1)$ and $\vec{Z}2(a2, b2, c2)$. Such a double solution is understandable in geometry, because \vec{Z} results from the intersection of a plane containing \vec{G} et \vec{z} , with the cone defined by axis B and aperture A .

It will be noted that Go, K and L are calculated from the measurements made by accelerometers $G1, G2$, magnetometers $B1, B2, B3$, while B and A are already known. Therefore, it is quite possible to calculate the values of λ .

For obtaining the value of the slope p , the vertical speed of the buoy will first be calculated by taking into account the values related to the rectangular-coordinate system defined by the root $\vec{Z}1$ of equation (5).

$$V_{Z1} = \int_0^+ \vec{Z}1 \cdot (\vec{G} - \vec{g}) dt$$

or:

$$V_{Z1} = \int_0^+ (a1 \cdot Gx + b1 \cdot Gy + c1 \cdot Gz + g) dt$$

because \vec{g} is oriented downwards, or also:

$$V_{Z1} = \int_0^+ [\lambda 1 \cdot Gx^2 + \lambda 1 \cdot Gy^2 - (L + \lambda 1 \cdot K) \cdot Gz + g] dt$$

or:

$$V_{Z1} = \int_0^+ [\lambda 1 (Go^2 - K \cdot Gz) - L \cdot Gz + g] dt \tag{6}$$

The equation (6) provides the sign of V_{Z1} and, on the other hand, $c1$ is equal to $-(L + \lambda K)$, therefore:

$$p1 = -(\text{sign } V_{Z1}) \text{ Arc cos } c1$$

It can be noted that $c1$ and $c2$ are always positive since the absolute value is lower than 90° . Furthermore, equations (4) and (1), also give the results:

$$c1 = (1 - \lambda 1^2 \cdot Go^2)^{\frac{1}{2}} \tag{5}$$

Of course, the last calculations on $\lambda 1$ are also convenient for calculating $\lambda 2$.

The projection $\vec{z}1$ of \vec{z} on the horizontal plane OX1, OY1, defined by $\vec{Z}1$, will now be calculated:

$$\vec{z}1 \cdot \vec{Z}1 = 0$$

$$(\vec{z} - \vec{z}1) \cdot \vec{Z}1 = 0$$

or, with $\vec{z}1(a1, \beta 1, \gamma 1)$,

$$a1 \cdot a1 + \beta 1 \cdot b1 + \gamma 1 \cdot c1 = 0 \tag{8}$$

$$a1 \cdot b1 - a1 \cdot \beta 1 = 0$$

$$a1(\gamma 1 - 1) - a1c1 = 0 \tag{9}$$

$$\beta 1c1 - b1(\gamma 1 - 1) = 0$$

which results in: