$$W' = \begin{bmatrix} W_1 & \dots & 0 & 0 \\ \dots & & & \dots \\ 0 & \dots & W_{i-1} & X \\ 0 & \dots & 0 & W_i' \end{bmatrix} \quad (1)$$

where X is a vector multiple of the last row of W_i. (If the transformations of W_i are linearly dependent on more than the transformations of W_{i-1} then there will be more nonzero entries in the last block column of W'.) A case II degeneracy will affect the grouping algorithm since the row-echelon form will now show a second rigid body dependent on an earlier rigid body. The algorithm can be modified to handle this case by first identifying non-base columns of the echelon form that have nonzeros corresponding to no more than four base columns. These correspond to points belonging to the W_{i-1} in the equation above. After these have been found the object corresponding to W_{i-1} has been identified. Now points dependent on a greater number of base columns can be identified to identify a second object.

If there is more than one dependency between two objects a similar argument shows that the above reduction operation (deleting a column of T, subtracting a multiple of a row of W from other rows, and deleting that row from W) will be repeated. As long as the result is a block upper triangular form, the modification suggested above can be used.

It is possible that a later dependency could cause a row from W_{i-1} to be subtracted from the last block, destroying the block upper triangular form. If that happens, it is necessary to reconsider the choice of pivot columns in the decomposition to echelon form. (This can be done starting with the current echelon form.) If the right choice of base columns can be found, the echelon form will be block upper triangular with block of more than 4 by 4. When larger blocks are found, it appears necessary to consider all combinations of pivot column choices. This is exponential in the size of the dependent blocks, so in the worst case could be exponential in the size of the problem.

In a practical situation, future frames may introduce new feature points not previously identified and others identified in earlier frames may disappear. The SVD approach of earlier authors require that such points be dropped completely. In the present invention an algorithm is described that can be applied essentially in real time (assuming a fast enough processor) processing each new group of frames as they are obtained.

Initially a sufficient quantity of frames must be observed to obtain sufficient data to make a preliminary allocation of points to objects. If any points are not present in all objects, they must be discarded. As new data is obtained, new points may be identified and earlier points disappear. Disappearance presents no problem: the columns are simply discarded from the matrix. If they are not base columns, nothing needs to be done. If they are base columns, another base column needs to be identified from the columns belonging to those points in the same object. This can be done by a echelon form reduction on the reduced matrix. It is not necessary to return to the original data. When new points are identified, they should be ignored until several frames have been observed. At that point, an echelon reduction can be performed using only those rows in which the new points have been observed. Since base columns have already been identified, all this reduction needs to do is to determine the dependencies of these new columns(points) in terms of existing base columns. If such dependencies are observed, the point can be grouped with an existing object. If none are observed, it is a candidate for a point in a new object.

Having grouped features or points with respective moving bodies in the image, the information is provided to an application specific apparatus for appropriate additional signal processing such as, for example, in collision avoidance or robot vision applications. Depending upon the specific application, the additional processing relies upon the point grouping to determine the trajectory of the moving body associated with the grouped points.

While there has been described the grouping of points or features associated with moving bodies in an image, it will be apparent to those skilled in the art that variations and modifications are possible without deviating from the broad principles and spirit of the present invention which shall be limited solely by the scope of the claims appended hereto.

What is claimed is:

1. An imaging system for determining the number of independent moving bodies in image frames containing a plurality of moving bodies, associating points in the image with respective bodies and determining the trajectories of the moving bodies comprising:

means for obtaining a sequence of images containing a plurality of moving bodies;

feature detector means for obtaining image data related to identified points arising from the moving bodies in said sequence of images;

computing means for receiving said image data related to identified points, said computing means:

forming a matrix W where column vector w_i of matrix W includes the coordinates of each ith point;

representing the set of all points of $M=\text{diag}(w_1,w_2, \dots ,w_s)$ where w_p, $p=1, \dots , s$ are point coordinates and there are s bodies and each body has a set of n_p feature points;

forming a block transformation matrix T having entries T_{jp},$j=1, \dots , m$ and $p=1, \dots , s$ where T_{jp} is the pth object in the jth frame subject to a transformation/projection;

representing the observed data, matrix D, as D=TM where the upper bound on the rank of D is 4 s and the lower bound is s where s is the number of independent moving bodies;

forming the echelon form of matrix D; and

providing output data commensurate with the number of independent bodies in the image and with which points are associated with respective bodies from the echelon form of matrix D where columns having non-zero elements in the same row correspond to points belonging to same rigid body if matrix T has full rank and where the rank of echelon form of matrix D is related to a lower bound on the number of independent bodies in the image, and

processor means receiving said output data for determining the trajectories of each moving body in said sequence of images.

2. An imaging system as set forth in claim **1**, where when matrix T is not of full rank, said forming a block transformation matrix T comprises deleting a column of matrix T, subtracting a multiple of a row of W from other rows and deleting that row from W.

3. An imaging system as set forth in claim **1**, wherein forming the echelon form of matrix D includes selecting a tolerance value.

4. An imaging system as set forth in claim **1**, wherein forming the echelon form of matrix D includes selecting a rank tolerance value and a grouping tolerance value.

5. An imaging system as set forth in claim **1**, wherein forming the echelon form of matrix D includes applying Gauss-Jordan elimination to rows of matrix D.